Adaptive Smooth Scattered-data Approximation for Terrain Modeling

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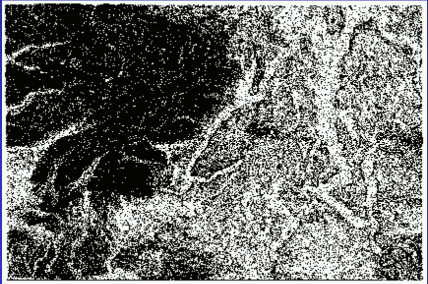
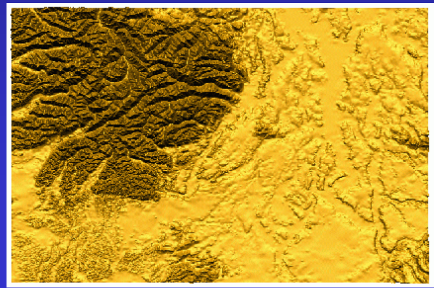
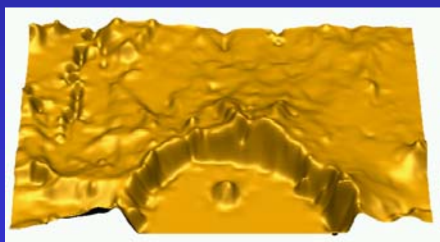


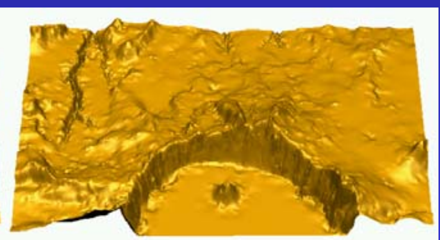


Contents

- Scattered-data Approximation
- Overview of Algorithm
- Local Fitting
- Merging Patches
- Results

Scattered-data Approximation





Adaptive Approximation

- highly non-uniform scattered data
- continuous approximations
  - error control
  - local refinement
- applications
  - visualization
  - compression / progressive transmission
  - surface reconstruction

Related Work

- radial basis functions
- splines
  - on triangles (box-, triangular splines)
  - on regular grids (B-splines)
  - on adaptive grids (hierarchical B-splines)
- least-squares fitting
  - large systems of equations
  - feasible for data on regular grids

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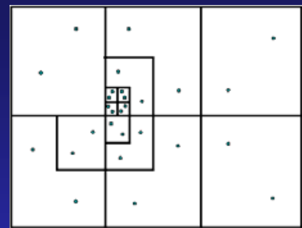
Overview of Algorithm

- adaptive clustering (quadtree)
- local fitting with polynomial patches
- merging patches to B-spline surface
- error estimation
- recursive refinement

Adaptive Clustering



- scattered data (p_i, f_i)
- uniform partitioning C_0
- for $(j=0,1,\ldots,n)$
  - continuous approximation $F_j(s,t)$
  - errors $\Delta f_i = f_i - F_j(p_i)$
  - refine clusters C_j with $\max \Delta f_i > \varepsilon$
  - replace $f_i := \Delta f_i$
- adaptive representation $\Sigma_j F_j(s,t)$

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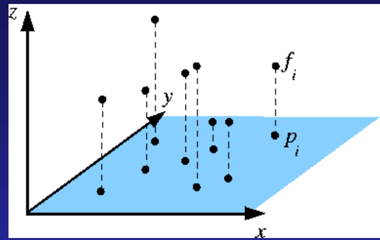
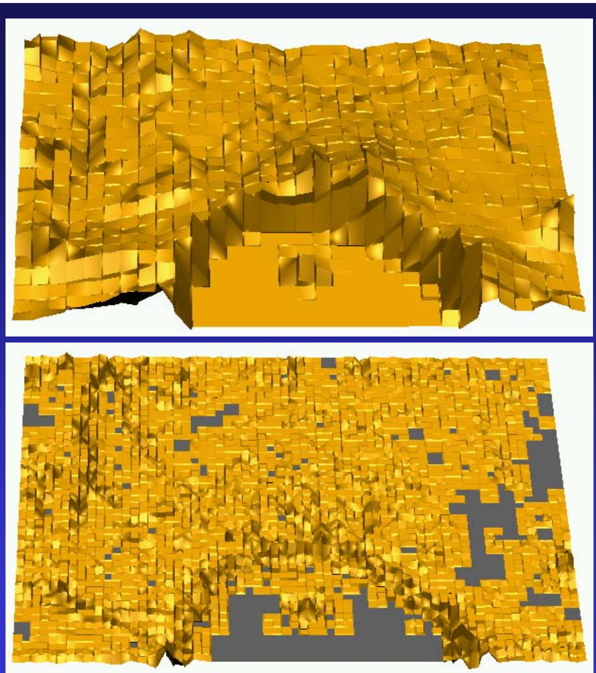
Local Fitting



- polynomial patches
  $P(s,t) = \Sigma_{j=1}^{n} c_j \phi_j (s,t)$
  (ϕ_j bivariate polynomials, deg=1,2)
- least-squares fitting
  $A^T A c = A^T f \qquad a_{ij} = \phi_j (p_i)$
- complexity $O(m\, n^2 + n^3)$
  m points, $n = 4,9$



Level j=0
(32 £ 24)

Level j=1
(64 £ 48)

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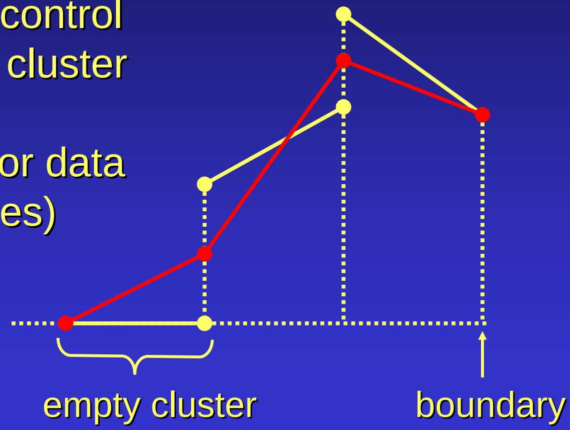
Merging Patches

- discontinuous B-spline representation with multiple knots (Bézier patches)
- reduce multiplicity by knot removal
  - smooth global representation (C^{deg-1})
  - support increases
  - add clusters at boundary of support

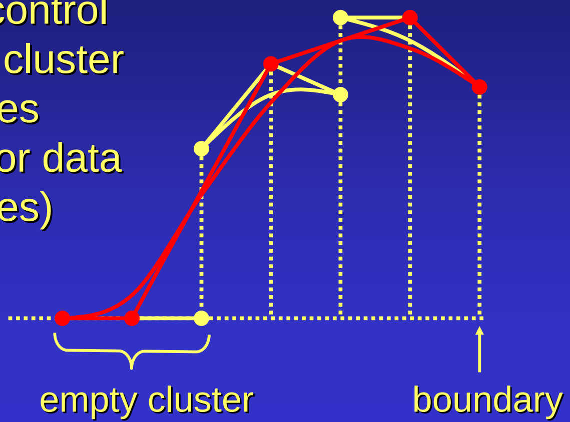
Bilinear Case

- average control points at cluster corners (except for data boundaries)

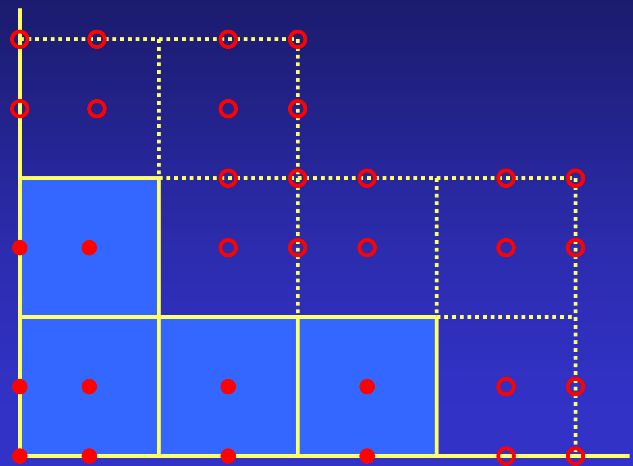


Biquadratic Case

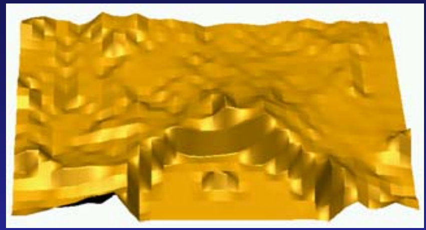
- remove control points at cluster boundaries (except for data boundaries)



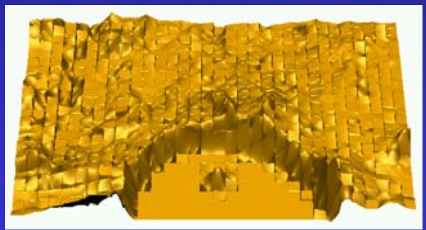
Biquadratic Case



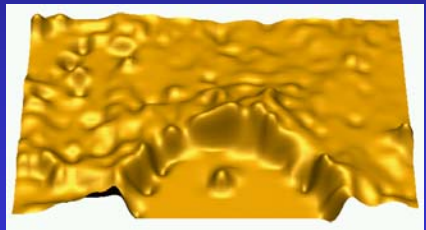
bilinear C^{-1}



bilinear C^{0}



biquadratic C^{-1}



biquadratic C^{1}

Efficient Evaluation

- level of detail m: $F(s,t) = \Sigma^{m}_{j=0} F_j(s,t)$
- for fixed level m:
  - match resolution by inserting knots on coarser levels $F_{0 \ldots} F_{m-1}$
  - add corresponding control points of $F_{0 \ldots} F_m$
  - evaluate only one representation, $F := F_m$

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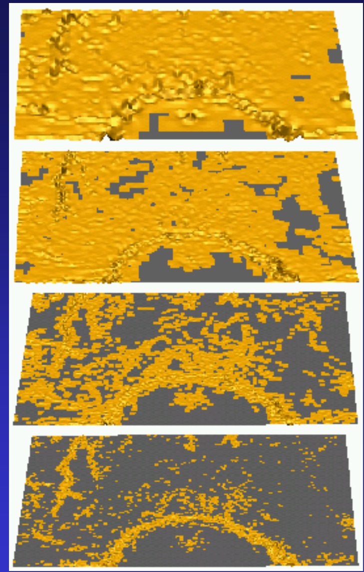
Results

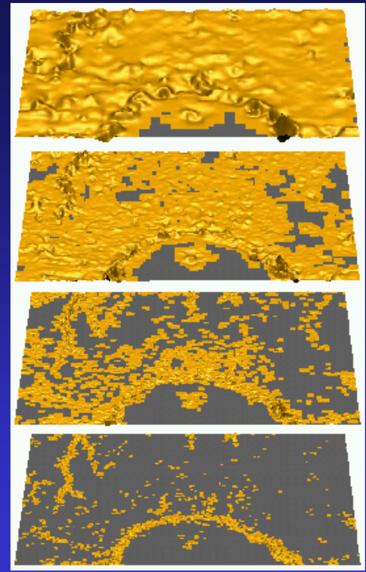
- data sets courtesy of USGS,
  - Crater Lake 18 800 points (11.8 %)
  - Seattle 587 000 points (0.33 %)
- downsampled, based on local curvature
- 6 levels computed, form 32£24 = 786 to 1024£786 = 786 400 clusters

Results

- Crater Lake
  - 0.26% L^2-error (threshold ε =0.5%)
  - 20.600 clusters on all levels (2.6% of 786 400)
  - 4.3 sec (3.3 for bilinear)
- Seattle
  - 0.79% L^2-error (threshold ε =1%)
  - 319.400 clusters on all levels (40.6%)
  - 23.1 sec (11.8 for bilinear)



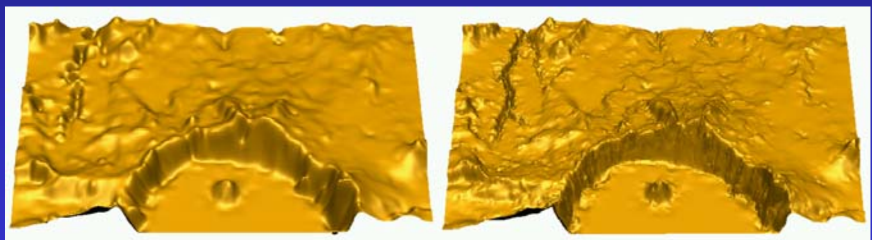
bilinear $F_1 \ldots F_4$



biquadratic $F_1 \ldots F_4$

Crater Lake

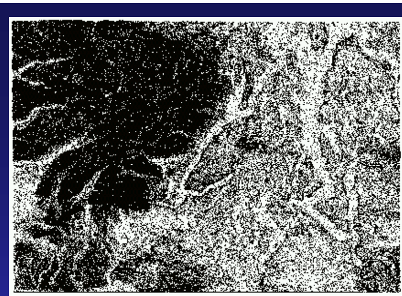
18 800 points

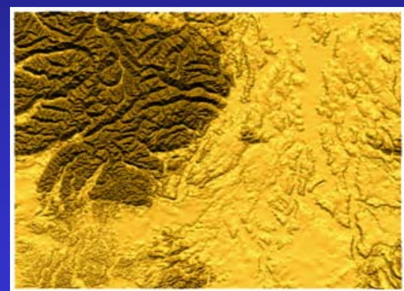


768 clusters, 4.65% L^2 ... 20 600 clusters, 0.26% L^2

Seattle

587 000 points





3 072 clusters, 13.3% L^2 ... 319 400 clusters, 0.79% L^2

Conclusions and Future Work

- selected scattered data provide basis for efficient continuous approximation
- problems in sparse regions near steep gradients
- feasible extensions
  - 3D domain (volumes)
  - 3D range (vector fields, FFD's)